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A WORLDWIDE CORRELATION FOR EXPONENTIAL BED PARTICLE SIZE VARIATION IN SUBAERIAL AQUEOUS FLOWS

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ABSTRACT

The particle size of the bed sediments in or on many natural streams, alluvial fans, laboratory flumes, irrigation canals and mine waste deltas varies exponentially with distance along the stream. A plot of the available worldwide exponential bed particle size diminution coefficient data against stream length is presented which shows that all the data lie within a single narrow band extending over virtually the whole range of stream lengths and bed sediment particle sizes found on Earth. This correlation applies to both natural and artificial flows with both sand and gravel beds, irrespective of either the solids concentration or whether normal or reverse sorting occurs. This strongly suggests that there are common mechanisms underlying the exponential diminution of bed particles in subaerial aqueous flows of all kinds. Thus existing models of sorting and abrasion applicable to some such flows may be applicable to others. A comparison of exponential laboratory abrasion and field diminution coefficients suggests that abrasion is unlikely to be significant in gravel and sand bed streams shorter than about 10 km to 100 km, and about 500 km, respectively. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: exponential sorting; fluvial abrasion; hydraulic sorting; mobile bed flow

INTRODUCTION

It has long been known that the particle size of the bed sediments of many natural streams flowing through their own sediments varies exponentially or nearly so with distance along the stream (Sternberg, 1875; Grabau, 1913). More recently it has been shown that the same exponential relationship applies to bed particles on alluvial fans (Bluck, 1964) and in laboratory flumes (Straub, 1935; Paola *et al.*, 1992), irrigation canals (Simons, 1977), and mine waste deltas (Williams and Morris, 1989; Morris and Williams, 1997a).

Various power and linear equations have also been used to model downstream fining (Bentel, 1981; Brierly and Hicken, 1985) and all fit well on occasion. However, exponential equations are, in general, the most appropriate (Yatsu, 1955; Morris and Williams, 1998). Moreover, they have a theoretical link to the particle size distributions of sediments that other equations lack (Morris, 1993; Morris and Williams, 1997b).

In this paper, a plot of the available worldwide exponential bed particle size diminution coefficient data for natural streams, alluvial fans, laboratory flumes, irrigation canals and mine waste deltas versus the corresponding stream segment length is presented. Despite their disparate origins, all of the data lie within a single narrow band extending over virtually the whole range of stream lengths and bed sediment particle sizes found on Earth.

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BED PARTICLE SIZE VARIATION CORRELATION

Bed particle size diminution equations

The changes with increasing distance downstream of the weight and size of particles on the bed of a subaerial aqueous flow are described by (Sternberg, 1875; Schoklitsch, 1933; Knighton, 1980):

$$W_x = W_0 \exp(-\phi x) \tag{1}$$

and

$$D_x = D_0 \exp(-\alpha x) \tag{2}$$

where x is the longitudinal coordinate relative to an arbitrary datum, ϕ and α are weight and size diminution coefficients, and W_x and D_x are the weight and size, respectively, of the characteristic bed particle at $x = x$. The coefficients ϕ and α both vary from stream to stream. For poorly sorted sediments, the characteristic particle size or weight may be taken as the median (Grabau, 1913; Williams and Morris, 1989; Morris, 1993) or maximum size or weight (Sternberg, 1875; Unrug, 1957; Bluck, 1964). Equations 1 and 2 apply to streams or stream segments that have reached equilibrium under steady or quasi-steady hydraulic conditions, with no significant lateral inflows of sediments or external controls such as rock bars or dams (Morris and Williams, 1997b).

The coefficients ϕ and α are conventionally related by (Mikos, 1995; Morris and Williams, 1997b):

$$\alpha = \frac{1}{3}\phi \tag{3}$$

Inverting Equation 2 gives:

$$\alpha = \ln\left(\frac{D_0}{D_x}\right)x^{-1} \tag{4}$$

Equation 4 shows that zero α values are obtained when D is constant over the length of the stream. This also implies a straight (constant slope) longitudinal stream profile (Morris and Williams, 1997b). Negative α values are obtained for reverse sorting, in which the bed particles increase in size in the downstream direction due to overpassing (Everts, 1973; Komar, 1987; Morris and Williams, 1997b).

Equivalent α for sediments with variable specific gravity

Equation 4 may be used to calculate α values for stream segments with sediments with constant specific gravity. However, in addition to particle size (Equation 2), sediments with variable specific gravity sort according to (Morris, 1993; Morris and Williams, 1997a):

$$(G_s - G_f)_x = (G_s - G_f)_0 \exp(-\beta x) \tag{5}$$

where β is a diminution coefficient and G_s and G_f are the specific gravities of the bed sediment and the transporting fluid, respectively. No convincing negative β values are known at present.

To account for sorting by G_s, an equivalent α, α_{eq}, was calculated based on the sorting of such sediments according to their dimensionless particle size, D_*, defined by (van Rijn, 1984):

$$D_* = D\left[\frac{(G_s - G_f)g}{\nu^2}\right]^{1/3} \tag{6}$$

where g is the acceleration due to gravity, and ν is the kinematic viscosity of water.

Equation 6 accounts for the G_s of different sediments, transporting fluids other than water (G_f, v), and the strength of the gravitational field causing the flow (g). However, this study was based on the familiar Equation 2 because the transporting fluid was water ($G_f = 1$) in all cases. The effect of fines in suspension on both v and G_f was ignored.

Combining Equations 2, 5 and 6 gives:

$$D_{*_x} = D_{*_0} \exp\left(-\alpha_{eq} x\right) \tag{7}$$

where

$$\alpha_{eq} = \alpha + \frac{\beta}{3} \tag{8}$$

If G_f is constant, $\beta = 0$ for sediments with constant G_s (Equation 5) and $\alpha_{eq} = \alpha$ (Equation 8). Hence the α_{eq} of sediments with variable G_s are directly comparable with the α of those with constant G_s.

Worldwide particle size diminution data

Data conforming to Equations 1, 2 and 7 were obtained for 60 natural streams or stream segments, 25 alluvial fans and micro-fans, four boulder jams, four irrigation canals, five mine tailings deltas, eight coal mine waste co-disposal deltas, and three laboratory flumes (Table I). In most cases, the equations were fitted to the data using least-squares methods. Where necessary, Equation 3 was used to convert ϕ values to α values. In the absence of other data, a G_s value of 2·65 (Bogardi, 1974; Ohmori, 1991) was assumed in converting W_0 to D_0. The D listed in Table I are median or, where indicated, maximum sizes. Of the data whose level of significance, p, was known or could be calculated (100 of 115 data points), only those with p values less than 10 per cent were accepted. Data with unquantified p values were accepted (15 data points) either on the basis of clear graphical evidence that their p values were less than 10 per cent or because they had been accepted previously as significant in analyses of exponential sorting (Bluck, 1964; Simons, 1977).

The natural stream data include data from recent rivers, creeks, arroyos, sandurs and glacial outwashes, and ancient stream gravel beds in North America, Japan, New Zealand, Great Britain and Europe. The alluvial fan data are from the United States and Great Britain, the irrigation canal data from Pakistan, and the mine waste data from South Africa, Canada and Australia.

Of all the sediments considered, only coal mine wastes have significantly variable G_s. Hence the calculation of α_{eq} values was necessary only for the coal tailings and coal co-disposal deltas. Since the β values for these materials are comparable to and often less than the associated α values, taken as positive (Morris, 1993; Morris and Williams, 1997a, c), their effect on the corresponding α_{eq} values is relatively small (Equation 8).

The stream or stream segment length, L, for boulder jams, mine waste deltas and laboratory flume tests is well defined due to the exhaustion of the bed load under a single hydraulic regime (Morris, 1993; Morris and Williams, 1997b). In the absence of significant lateral sediment inflows and external controls, this is also true of alluvial fans and the upstream segments of streams in which a gravel to sand bed transition occurs (Yatsu, 1955). For other stream segments and for canals, the determination of L is more problematical. It is often uncertain whether the bed load is exhausted within the available L. The associated α values give no guidance because they are (theoretically) independent of L (Equations 2 and 4). Some L values in the present data set are determined by external controls, such as rock bars, dams and major tributaries or distributaries, which initiate new hydraulic or sediment regimes. In the absence of such controls, the longest L was accepted for which a p value of less than 10 per cent was obtained for Equation 1 or 2. Thus some L values in the present data set may be underestimated significantly.

Table I. Bed particle size and sorting data for natural stream segments, alluvial fans, irrigation canals, mine tailings and co-disposal deltas, and laboratory flumes

	Length (km)	D (mm)	D_0 (mm)	α (m^{-1})	p (%)	References
Natural streams						
Mississippi River	1770·0	0·12–0·72	–	$8·51 \times 10^{-7}$	–	Rafay in Simons (1977)
Mississippi River	1710·0	0·14–0·67	$5·22 \times 10^{-1}$	$6·11 \times 10^{-7}$	0·3	Russell and Taylor (1937)
South Canadian River	1006·0	0·082–0·29	$2·17 \times 10^{-1}$	$4·89 \times 10^{-7}$	1·4	Pollack (1961)
Peace River	469·9	15–50	$4·80 \times 10^{1}$	$2·04 \times 10^{-6}$	8·5	Kellerhals *et al.* (1972)
Peace River	410·1	0·14–0·35	$3·55 \times 10^{-1}$	$1·56 \times 10^{-6}$	4·1	Kellerhals *et al.* (1972)
North Saskatchewan River	393·0	7–28	$2·17 \times 10^{1}$	$2·24 \times 10^{-6}$	<0·04	Kellerhals *et al.* (1972)
Colorado River granitics*	350·8	20–124	$1·09 \times 10^{2}$	$3·78 \times 10^{-6}$	<0·04	Bradley (1970)
Colorado River chert*	350·8	43–169	$1·46 \times 10^{2}$	$2·10 \times 10^{-6}$	0·1	Bradley (1970)
Colorado River quartz*	350·8	26–124	$1·11 \times 10^{2}$	$2·60 \times 10^{-6}$	<0·04	Bradley (1970)
Clutha River quartz*	300·0	50–400	$3·26 \times 10^{2}$	$2·88 \times 10^{-6}$	–	Adams (1979)
Clutha River chlorite schist*	300·0	30–1100	$1·02 \times 10^{3}$	$9·65 \times 10^{-6}$	–	Adams (1979)
Oldman River	282·4	16–39	$3·72 \times 10^{1}$	$3·05 \times 10^{-6}$	2·5	Kellerhals *et al.* (1972)
Red Deer River	266·2	16–112	$6·01 \times 10^{1}$	$3·01 \times 10^{-6}$	1·9	Kellerhals *et al.* (1972)
Middle Rhine*	260·9	42–162	$1·74 \times 10^{2}$	$4·83 \times 10^{-6}$	<0·04	Sternberg (1875); Barrell (1925) Schoklitsch (1937)
North Saskatchewan River	258·6	21–130	$1·04 \times 10^{2}$	$4·94 \times 10^{-6}$	3·7	Kellerhals *et al.* (1972)
Rio Grande	241·4	0·14–0·50	–	$3·67 \times 10^{-6}$	–	Rafay in Simons (1977)
Dunajec River*	192·0	45–555	$6·07 \times 10^{2}$	$1·11 \times 10^{-5}$	0·2	Unrug (1957)
Peace River	140·0	–	$6·34 \times 10^{1}$	$3·35 \times 10^{-6}$	<0·5	Church and Kellerhals(1978)
Iller River	126·0	97–136	$1·36 \times 10^{2}$	$2·86 \times 10^{-6}$	0·06	Hildebrand in Schoklitsch (1937)
Mur River	120·0	34–75	$7·56 \times 10^{1}$	$6·10 \times 10^{-6}$	<0·04	Hochenburger in Grabau (1913)
Peace River	90·0	–	$6·62 \times 10^{1}$	$4·94 \times 10^{-6}$	<2·5	Church and Kellerhals (1978)
Rubicon River*	78·7	457–3290	$2·30 \times 10^{3}$	$2·11 \times 10^{-5}$	<0·04	Scott and Gravlee (1968)
Kinu River	53·6	20–73	$6·84 \times 10^{-1}$	$2·57 \times 10^{-5}$	1·8	Yatsu (1955)
Peace River	50·0	–	$5·71 \times 10^{1}$	$9·75 \times 10^{-6}$	<1·0	Church and Kellerhals (1978)
Rapid Creek	48·0	5·1–24	$2·05 \times 10^{1}$	$2·53 \times 10^{-5}$	<0·04	Plumley (1948)
Kinu River	46·0	0·41–0·86	$8·07 \times 10^{-1}$	$2·15 \times 10^{-5}$	6·9	Yatsu (1955)
Standing Stone Creek	43·5	12–117	$9·44 \times 10^{1}$	$3·78 \times 10^{-5}$	0·4	Brush (1961)
Kiso River	40·0	0·40–0·54	$5·37 \times 10^{-1}$	$7·08 \times 10^{-6}$	7·1	Yatsu (1955)
Nagara River	40·0	0·75–1·1	1·15	$1·42 \times 10^{-5}$	1·2	Yatsu (1955)
Yahagi River	35·0	1·0–2·0	1·89	$2·15 \times 10^{-5}$	<0·04	Yatsu (1955)
Bear Butte Creek	33·8	3·7–52	$3·53 \times 10^{1}$	$6·76 \times 10^{-5}$	0·06	Plumley (1948)
Upper Rhine	32·0	76–147	$1·48 \times 10^{2}$	$1·99 \times 10^{-5}$	<0·04	Collett in Schoklitsch (1937)
Maryland Upland Gravels	30·4	4·1–32	$3·2 \times 10^{1}$	$3·73 \times 10^{-5}$	–	Schlee (1957)
Honey Creek	28·2	31–91	$8·06 \times 10^{1}$	$2·95 \times 10^{-5}$	2·1	Brush (1961)
Battle Creek	28·0	5·3–33	$1·59 \times 10^{1}$	$5·07 \times 10^{-5}$	7·7	Plumley (1948)
Bollin River	26·0	7·4–67	$7·43 \times 10^{1}$	$8·28 \times 10^{-5}$	0·16	Knighton (1980)
Knik River*	25·7	41–327	$3·27 \times 10^{2}$	$8·08 \times 10^{-5}$	–	Bradley *et al.* (1972)
Abe River	23·0	17–83	$8·36 \times 10^{1}$	$6·48 \times 10^{-5}$	<0·04	Yatsu (1955)
Dean River	22·6	14–70	$7·96 \times 10^{1}$	$7·1 \times 10^{-5}$	5·4	Knighton (1980)
Watarase River	21·3	32–75	$6·40 \times 10^{1}$	$4·62 \times 10^{-5}$	0·5	Yatsu (1955)
Squamish River	21·0	53–236	$1·68 \times 10^{2}$	$5·3 \times 10^{-5}$	<0·04	Brierly and Hickin (1985)
Shaver Creek	20·6	23–141	$1·10 \times 10^{2}$	$7·49 \times 10^{-5}$	<0·04	Brush (1961)
Hii River	20·0	0·9–2·5	–	$2·4 \times 10^{-5}$	–	Yatsu in Simons (1977)

Table I. Continued

	Length (km)	D (mm)	D_0 (mm)	α (m^{-1})	p (%)	References
Sho River	20·0	29–48	$4{\cdot}76 \times 10^{1}$	$2{\cdot}95 \times 10^{-5}$	<0·04	Yatsu (1955)
Noe River all sediments	19·4	30–70	$6{\cdot}86 \times 10^{1}$	$4{\cdot}2 \times 10^{-5}$	0·2	Knighton (1980, 1982)
Noe River sandstone	19·4	35–95	$8{\cdot}5 \times 10^{1}$	$4{\cdot}6 \times 10^{-5}$	–	Knighton (1982)
Noe River shale	19·4	20–60	$3{\cdot}9 \times 10^{1}$	$1{\cdot}6 \times 10^{-5}$	–	Knighton (1982)
Gillis Falls	17·7	7·1–80	$1{\cdot}37 \times 10^{1}$	$-1{\cdot}08 \times 10^{-4}$	0·4	Hack (1957)
Kiso River	15·6	39–66	$6{\cdot}62 \times 10^{1}$	$3{\cdot}08 \times 10^{-5}$	7·2	Yatsu (1955)
Bollin River	14·9	0·33–1·1	1·17	$8{\cdot}88 \times 10^{-5}$	9·3	Knighton (1980)
Tenryu River	14·9	24–49	$4{\cdot}68 \times 10^{1}$	$4{\cdot}44 \times 10^{-5}$	5·8	Yatsu (1955)
Arroyo Seco*	14·8	460–1830	$1{\cdot}56 \times 10^{3}$	$8{\cdot}75 \times 10^{-5}$	<0·04	Krumbein (1942)
North River	14·2	10–55	$5{\cdot}73 \times 10^{1}$	$1{\cdot}23 \times 10^{-4}$	0·9	Hack (1957)
Weiker Run	13·4	95–142	$1{\cdot}39 \times 10^{2}$	$2{\cdot}21 \times 10^{-5}$	7·8	Brush (1961)
Nagara River	13·0	26–41	$3{\cdot}98 \times 10^{1}$	$4{\cdot}21 \times 10^{-5}$	4·3	Yatsu (1955)
Copper River Delta	12·0	30–300	$3{\cdot}00 \times 10^{2}$	$1{\cdot}92 \times 10^{-4}$	–	Boothroyd (1970)
Tye River	10·6	230–630	$6{\cdot}10 \times 10^{2}$	$8{\cdot}38 \times 10^{-5}$	4·6	Hack (1957)
Sunwapta River	8·3	6·1–82	$7{\cdot}52 \times 10^{1}$	$2{\cdot}18 \times 10^{-4}$	<0·04	Dawson (1988)
Makita River	7·0	100–200	–	$1{\cdot}1 \times 10^{-4}$	–	Yatsu in Simons (1977)
Reeds Run	5·2	30–86	$6{\cdot}53 \times 10^{1}$	$1{\cdot}87 \times 10^{-4}$	3·1	Brush (1961)
Arroyo Larguito	2·6	0·20–0·52	$2{\cdot}11 \times 10^{-1}$	$-2{\cdot}87 \times 10^{-4}$	6·4	Woodford (1951)
Allt Dubhaig	2·3	20–98	$7{\cdot}15 \times 10^{1}$	$5{\cdot}97 \times 10^{-4}$	0·3	Ferguson and Ashworth (1991)
Sunwapta River	2·2	0·2–57	$6{\cdot}29 \times 10^{1}$	$6{\cdot}92 \times 10^{-4}$	<0·04	Dawson (1988)
Lewis Sandur	1·0	5·5–19	$1{\cdot}55 \times 10^{1}$	$1{\cdot}26 \times 10^{-3}$	2·7	Church (1972)
Peyto Glacier Outwash*	0·94	105–293	$3{\cdot}26 \times 10^{2}$	$1{\cdot}14 \times 10^{-3}$	<0·04	McDonald and Banerjee (1971)
Bow Glacier Outwash*	0·68	116–224	$2{\cdot}20 \times 10^{2}$	$9{\cdot}30 \times 10^{-4}$	0·5	McDonald and Banerjee (1971)
Alluvial fans						
Trail Canyon	16·6	7·4–74	$3{\cdot}51 \times 10^{1}$	$5{\cdot}53 \times 10^{-5}$	7·9	Denny (1965)
Eagle Mountain	15·9	3·4–12	$1{\cdot}01 \times 10^{1}$	$6{\cdot}41 \times 10^{-5}$	5·1	Denny (1965)
Hanaupa Canyon	14·6	11–45	$4{\cdot}57 \times 10^{1}$	$4{\cdot}18 \times 10^{-5}$	8·6	Denny (1965)
Johnson Canyon	10·2	15–32	$2{\cdot}82 \times 10^{1}$	$6{\cdot}20 \times 10^{-5}$	0·8	Denny (1965)
Shadow Mountain	10·0	2·3–98	$4{\cdot}85 \times 10^{1}$	$3{\cdot}11 \times 10^{-4}$	<0·04	Denny (1965)
Shadow Mountain	10·0	4·0–23	$2{\cdot}22 \times 10^{1}$	$1{\cdot}49 \times 10^{-4}$	0·07	Denny (1965)
Alkali Flat	9·2	2·4–52	$3{\cdot}25 \times 10^{1}$	$2{\cdot}05 \times 10^{-4}$	<0·04	Denny (1965)
Lytle Creek*	8·1	762–2210	$2{\cdot}25 \times 10^{3}$	$1{\cdot}38 \times 10^{-4}$	3·9	Eckis (1928)
Shadow Mountain	7·3	6·5–28	$2{\cdot}65 \times 10^{1}$	$2{\cdot}01 \times 10^{-4}$	<0·04	Denny (1965)
Santa Catalina*	6·4	184–4040	$2{\cdot}52 \times 10^{3}$	$4{\cdot}27 \times 10^{-4}$	<0·04	Blissenbach (1952)
Arrow River Canyon*	5·5	434–1133	$1{\cdot}13 \times 10^{-3}$	$2{\cdot}34 \times 10^{-4}$	<0·04	Bluck (1964)
Bat Mountain	5·4	2·6–14	$1{\cdot}38 \times 10^{1}$	$2{\cdot}39 \times 10^{-4}$	<0·04	Denny (1965)
Bat Mountain	4·3	4·9–16	$1{\cdot}32 \times 10^{1}$	$1{\cdot}84 \times 10^{-4}$	0·5	Denny (1965)
Deadman Pass	3·9	3·4–15	$1{\cdot}56 \times 10^{1}$	$4{\cdot}01 \times 10^{-4}$	<0·04	Denny (1965)
Antelope Springs*	3·7	30–884	$1{\cdot}26 \times 10^{3}$	$6{\cdot}42 \times 10^{-4}$	0·08	Lustig (1965)
Shadow Mountain	3·4	4·0–30	$1{\cdot}97 \times 10^{1}$	$6{\cdot}99 \times 10^{-4}$	<0·04	Denny (1965)
La Crescenta*	2·5	615–2770	$2{\cdot}56 \times 10^{3}$	$5{\cdot}65 \times 10^{-4}$	<0·04	Chawner in Krumbein (1942)
Aubrey Cliffs B*	1·6	111–854	$1{\cdot}02 \times 10^{3}$	$1{\cdot}13 \times 10^{-3}$	<0·04	Blissenbach (1952)
Aubrey Cliffs A*	0·64	143–900	$9{\cdot}53 \times 10^{2}$	$2{\cdot}96 \times 10^{-3}$	1·8	Blissenbach (1952)
Bat Mountain	0·64	4·6–13	$1{\cdot}00 \times 10^{1}$	$5{\cdot}82 \times 10^{-4}$	0·6	Denny (1965)

Table I. Continued

	Length (km)	D (mm)	D_0 (mm)	α (m^{-1})	p (%)	References
Paiute Chute*	0·37	914–2990	$2{\cdot}47 \times 10^{3}$	$1{\cdot}53 \times 10^{-3}$	2·9	Lustig (1965)
Paiute Chute*	0·30	884–3170	$2{\cdot}39 \times 10^{3}$	$2{\cdot}82 \times 10^{-3}$	2·1	Lustig (1965)
Sker Point*	0·16	117–401	$3{\cdot}74 \times 10^{2}$	$7{\cdot}46 \times 10^{-3}$	<0·04	Bluck (1965)
Westgard Pass A*	0·017	91–274	$2{\cdot}55 \times 10^{2}$	$5{\cdot}09 \times 10^{-2}$	0·06	Lustig (1965)
Westgard Pass B*	0·014	61–244	$2{\cdot}39 \times 10^{2}$	$8{\cdot}71 \times 10^{-2}$	0·4	Lustig (1965)
Boulder jams						
Arroyo Seco Site 17	0·11	182–274	$1{\cdot}93 \times 10^{2}$	$-2{\cdot}93 \times 10^{-3}$	<0·04	Krumbein (1942)
Arroyo Seco Site 21	0·061	97–556	$8{\cdot}88 \times 10^{1}$	$-2{\cdot}73 \times 10^{-2}$	<0·04	Krumbein (1942)
Arroyo Seco Site 25B	0·053	101–148	$1{\cdot}08 \times 10^{2}$	$-5{\cdot}42 \times 10^{-3}$	0·2	Krumbein (1942)
Arroyo Seco Site 25A	0·046	137–504	$1{\cdot}39 \times 10^{2}$	$-2{\cdot}65 \times 10^{-2}$	<0·04	Krumbein (1942)
Irrigation canals						
Morali-Ravi	79·2	0·11–0·32	–	$8{\cdot}70 \times 10^{-6}$	–	Simons (1977)
Dipalpur	54·9	0·09–0·17	–	$8{\cdot}26 \times 10^{-6}$	–	Simons (1977)
Upper Chenab	36·6	0·12–0·27	–	$1{\cdot}52 \times 10^{-5}$	–	Simons (1977)
B. S.	21·3	0·10–0·26	–	$2{\cdot}78 \times 10^{-5}$	–	Simons (1977)
Tailings deltas						
Denison uranium	0·518	0·050–0·13	$2{\cdot}43 \times 10^{-1}$	$4{\cdot}68 \times 10^{-3}$	4·5	Conlin (1989)
Meandu coal**	0·180	0·018–0·028	$2{\cdot}59 \times 10^{-2}$	$1{\cdot}62 \times 10^{-3}$	<0·04	Morris (1993)
Diamond	0·120	1·7–4·8	1·50	$-8{\cdot}83 \times 10^{-3}$	0·7	Blight (1994)
Platinum Al	0·115	0·070–0·20	$1{\cdot}77 \times 10^{-1}$	$1{\cdot}18 \times 10^{-2}$	2·9	Bentel (1981)
Aberdare coal**	0·070	0·066–0·35	$3{\cdot}61 \times 10^{-1}$	$3{\cdot}20 \times 10^{-2}$	<0·04	Morris (1993)
Coal co-disposal deltas						
Jeebropilly**	0·123	3·0–21	3·25	$-1{\cdot}49 \times 10^{-2}$	0·7	Morris and Williams (1997c)
Goonyella-Riverside 6**	0·030	5·0–20	3·61	$-5{\cdot}18 \times 10^{-2}$	0·3	Morris and Williams (1997a)
Goonyella-Riverside 3**	0·030	3·5–20	3·24	$-5{\cdot}73 \times 10^{-2}$	0·04	Morris and Williams (1997a)
Goonyella-Riverside 4**	0·029	5·3–16	4·88	$-2{\cdot}70 \times 10^{-2}$	6·4	Morris and Williams (1997a)
Goonyella-Riverside 2**	0·024	4·3–13	4·79	$-3{\cdot}50 \times 10^{-2}$	4·3	Morris and Williams (1997a)
Warkworth 3**	0·023	5·9–15	5·68	$-3{\cdot}42 \times 10^{-2}$	8·7	Morris and Williams (1997a)
Goonyella-Riverside 1**	0·023	5·0–10	5·34	$-1{\cdot}92 \times 10^{-2}$	7·3	Morris and Williams (1997a)
Goonyella-Riverside 7**	0·018	3·9–8·4	2·96	$-4{\cdot}57 \times 10^{-2}$	0·7	Morris and Williams (1997a)
Laboratory flumes						
Seal 22 hrs*	0·027	21–43	$4{\cdot}53 \times 10^{1}$	$2{\cdot}31 \times 10^{-2}$	<0·04	Seal *et al.* (1997)
Paolo*	0·025	26–46	$4{\cdot}54 \times 10^{1}$	$2{\cdot}36 \times 10^{-2}$	<0·04	Paola *et al.* (1992)
Straub	0·0073	2·6–4·8	2·56	$-9{\cdot}39 \times 10^{-2}$	<0·04	Straub (1935)

* Maximum particle size
** α_{eq}

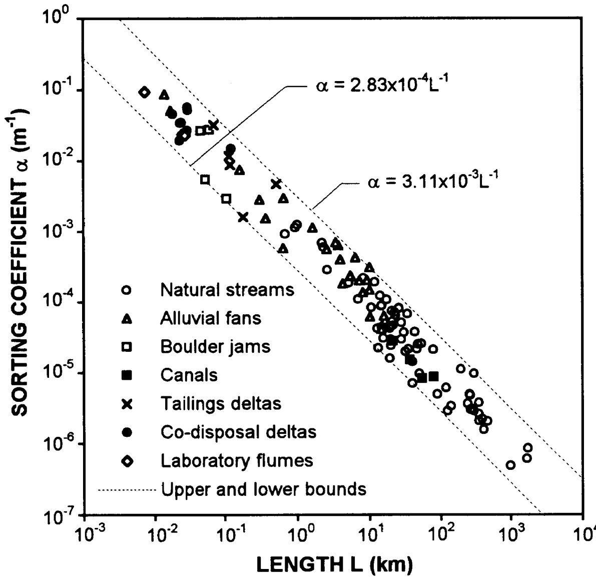

Figure 1. The variation of the particle size diminution coefficient α with the stream segment length L for worldwide subaerial aqueous flows of all kinds

The worldwide correlation

Figure 1, a plot of α versus L for worldwide data from subaerial aqueous flows of all types, with α taken as positive for both reverse and normal sorting, shows that bed particle sorting in such flows conforms to:

$$\alpha = AL^{-1} \tag{9}$$

where A is a constant. The linear correlation coefficient R and p for the data are $-0{\cdot}980$ and less than 0·04 per cent, respectively, indicating a very strong correlation.

In Figure 1, L ranges from 7·3 m to 1770 km (Table I). The latter value compares well (Figure 1) with that of 4265 km for the Ob River, Siberia, the longest stream segment on Earth not intersected by major tributaries (Vianna, 1979).

The upper and lower bounds of the data points are surprisingly well defined (Figure 1). The width of the data band is 1·04 orders of magnitude, and the geometric midrange value of A in Equation 9 is $9{\cdot}38 \times 10^{-4}$ for α and L expressed in m^{-1} and km, respectively.

Putting x equal to L in Equation 4 and combining the result with Equation 9 to eliminate L gives:

$$\ln\left(\frac{D_0}{D_L}\right) = A \tag{10}$$

Combining this and the upper and lower bound equations (Figure 1) shows that, for the present data set, D_0/D_L ranges from 1·3 to 22. That is, there is relatively little change in D over L in any of the flows in the data set.

Homogeneity of the data

The distributions of p for the individual data points based on Equations 1, 2 and 7, of normal and reverse sorting, of sand and gravel beds, and of the data points based on the median and maximum D are shown in Figure 2a, b, c and d, respectively. Here, sand and gravel size bed particles are respectively defined as those smaller and larger than 2 mm in size. Thus, for the present data set, they comprise sand and silt down to

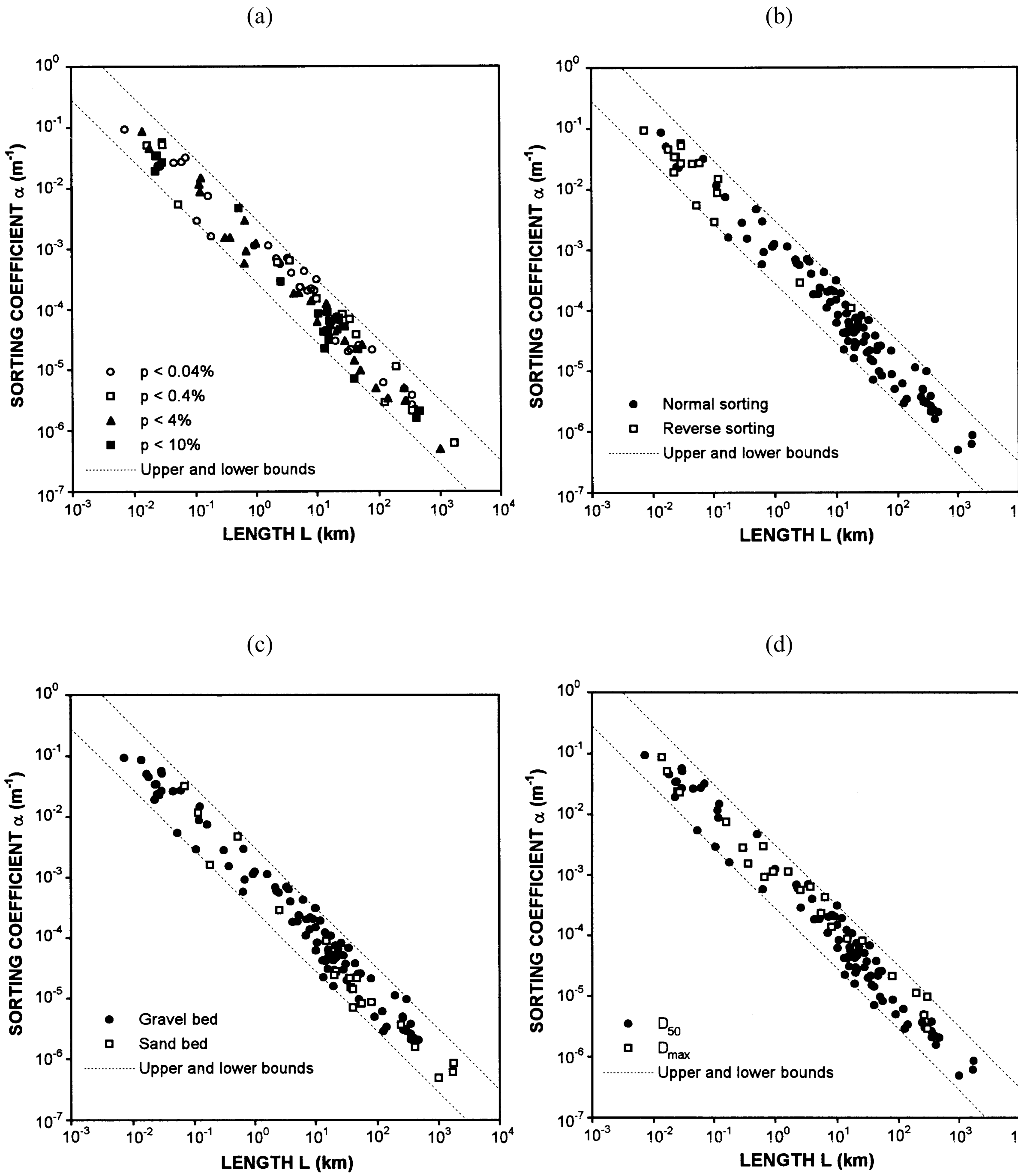

Figure 2. The variation of the particle size diminution coefficient α with the stream segment length L, showing the distributions of (a) the level of significance p for each data point, (b) normal and reverse sorting, (c) sand and gravel beds, and (d) correlations based on the median D_{50} and maximum D_{max} bed particle sizes

0·015 mm in size, and gravel, cobbles and boulders up to 0·63 m and 4·0 m in size for D_{50} and D_{max}, respectively. The corresponding range of D_* is from 0·40 to 15 400 and 102 000.

In Figure 2a to c, there are no obvious trends in the data relative to the upper and lower bounds. Hence the data set is probably homogeneous with respect to the corresponding parameters. Similar results were obtained for the distributions of the average and maximum bed slopes and of D_0 (Equation 2). These parameters ranged in magnitude from $1{\cdot}0 \times 10^{-4}$ to $4{\cdot}0 \times 10^{-1}$, $1{\cdot}1 \times 10^{-4}$ to $9{\cdot}1 \times 10^{-1}$, and 0·026 mm to 2·6 m, respectively.

All of the reverse sorting points (Figure 2b) were drawn from short streams (Table I) in which abrasion is negligible (Morris and Williams, 1997b). In both normal and reverse sorting in such streams, α is determined by the initial particle size distribution of the bed sediment and by L (Morris and Williams, 1997b). The mechanics of normal and reverse sorting differ significantly (Fenton and Abbott, 1977; Everts, 1973; Carling, 1987), but the present data suggest that, apart from the order of deposition, this has little effect on α (Figures 1 and 2b).

In Figure 2d, the D_{max} data points, which all correspond to rivers or alluvial fans, appear to be biased towards the upper bound, particularly for large L. This may be attributable to the wider range of particle sizes and hence higher α values (Equation 4) possible for D_{max} in comparison to D_{50}. However, it may also be an artefact of the relatively small D_{max} data set. It thus remains uncertain whether the data set is homogeneous with respect to D_{max}.

For the tailings deltas, co-disposal deltas and laboratory flumes, limited data on the input solids concentrations by weight C, discharges Q, and particle size distributions were available (6, 10 and 14 data points, respectively). The C values range from 0·0018 to 0·50, considerably higher than for most natural streams. The Q values range over only 0·05–0·14 m^3s^{-1}, while the ratios D_{75}/D_{25} of the upper and lower quartiles of the particle size distributions range from 5·8 to 70. The data points for C, Q and D_{75}/D_{25} cover almost the whole range between the upper and lower bounds and show no apparent trends with magnitude relative to the bounds. Hence, on the basis of the limited available data, the data set for the particle size diminution correlation appears to be homogeneous with respect to these parameters also.

The flows in natural streams, on coal co-disposal deltas, and in the gravel bed flumes and irrigation canals included in the data set are almost certainly all turbulent, but flows on tailings deltas may be laminar. The data point for the single almost certainly laminar flow (for platinum tailings with a C of 0·50) included in the data set lies close to the middle of the band of data points. However, considerably more laminar flow data are clearly required to enable any associated biases to be detected.

Thus, on the whole, all of the data appear to be reasonably homogeneous with respect to many of the parameters relevant to particle size diminution, with the possible exception of D_{max}. Hence it is reasonable to treat them, as has been done here, as a single set.

Significance of abrasion

As well as hydraulic sorting, Equations 1 and 2 also describe the reduction in the size of bed particles by abrasion and solution as they move downstream (Sternberg, 1875; Schoklitsch, 1933; Knighton, 1980). Hence Equation 4 may be used to calculate the abrasion coefficient, α_1.

Laboratory α_1 values for sand and gravel size particles are listed in Table II in order of decreasing maximum α_1. For the sand size particles, this corresponds to neither the order of increasing mineral hardness, H, according to Moh's scale (Table II), nor that of decreasing psephicity, defined as $(G_s-1)/H$ for aqueous sediment transport (Mackie, 1899). These inconsistencies reflect the variety of experimental techniques used to determine α_1, and similar, larger inconsistencies can be expected in the gravel data. The particularly high maximum α_1 values for gravel size andesite, chert, shale and coal listed in Table II were derived from high velocity abrasion and pipeline pumping tests.

Laboratory-derived α_1 values do not reflect all of the conditions and processes that occur in the field, such as abrasion in place (Schumm and Stevens, 1973) or clast weathering (Bradley, 1970). However, they may be regarded as at least indicative of field α_1 values. A comparison of the α and α_1 values listed in Tables I and II suggests that abrasion is unlikely to be significant in gravel bed streams shorter than about 10 km to 100 km, depending on the lithology of the bed sediments, or in sand bed streams shorter than about 500 km. This is broadly consistent with the suggestion by Morris and Williams (1997b), on the basis of limited field abrasion

Table II. Moh's hardness H and fluvial abrasion coefficient α_1 of minerals and rocks

Rock or mineral	H	Maximum α_1 (m^{-1})	Minimum α_1 (m^{-1})	Reference
Sand				
Limestone	3–4	$4\cdot3 \times 10^{-7}$	$9\cdot1 \times 10^{-9}$	Kuenen (1959)
Apatite	5	$2\cdot7 \times 10^{-7}$	$1\cdot4 \times 10^{-7}$	Thiel (1940)
Hornblende	5–6	$2\cdot5 \times 10^{-7}$	$1\cdot2 \times 10^{-7}$	Thiel (1940)
Garnet	6–7·5	$7\cdot8 \times 10^{-8}$	$5\cdot7 \times 10^{-8}$	Thiel (1940)
Quartz	7	$7\cdot7 \times 10^{-8}$	$2\cdot3 \times 10^{-10}$	Thiel (1940); Kuenen (1959); Schubert (1964)
Orthoclase	6	$5\cdot1 \times 10^{-8}$	$1\cdot3 \times 10^{-9}$	Kuenen (1959); Schubert (1964)
Gravel				
Andesite	–	$2\cdot0 \times 10^{-4}$	$1\cdot4 \times 10^{-6}$	Kuenen (1956); Kodama (1994)
Chert	–	$1\cdot1 \times 10^{-4}$	$2\cdot0 \times 10^{-6}$	Kodama (1994)
Shale	–	$9\cdot1 \times 10^{-5}$	–	Gies and Geller (1982)
Coal	–	$8\cdot3 \times 10^{-5}$	$2\cdot3 \times 10^{-6}$	Shook *et al.* (1979)
Limestone	3–4	$1\cdot0 \times 10^{-5}$	$1\cdot7 \times 10^{-7}$	Wentworth (1919); Schoklitsch (1937); Krumbein (1941); Kuenen (1956)
Gneiss	–	$6\cdot9 \times 10^{-6}$	$5\cdot8 \times 10^{-7}$	Schoklitsch (1937); Kuenen (1956); Shaw and Kellerhals (1982)
Granite	–	$4\cdot2 \times 10^{-6}$	$9\cdot0 \times 10^{-8}$	Wentworth (1919); Schoklitsch (1937); Kuenen (1956)
Obsidian	–	$3\cdot3 \times 10^{-6}$	$2\cdot7 \times 10^{-7}$	Kuenen (1956)
Greywacke	–	$1\cdot3 \times 10^{-6}$	$1\cdot4 \times 10^{-7}$	Kuenen (1956); Bradley *et al.* (1972); Shaw and Kellerhals (1982)
Amphibolite	4–6·5	$1\cdot2 \times 10^{-6}$	$6\cdot7 \times 10^{-7}$	Schoklitsch (1937)
Quartz	7	$1\cdot1 \times 10^{-6}$	$8\cdot7 \times 10^{-8}$	Schoklitsch (1937); Kuenen (1956); Bradley *et al.* (1972)
Aplite/pegmatite	6–7	$6\cdot8 \times 10^{-7}$	$1\cdot2 \times 10^{-7}$	Bradley (1970)
Quartzite	–	$5\cdot0 \times 10^{-7}$	$5\cdot7 \times 10^{-8}$	Kuenen (1956)
Quartz porphyry	–	$4\cdot7 \times 10^{-7}$	$7\cdot0 \times 10^{-8}$	Kuenen (1956)
Flint	–	$2\cdot3 \times 10^{-7}$	$2\cdot0 \times 10^{-7}$	Kuenen (1956)
Agate	–	$1\cdot7 \times 10^{-7}$	$9\cdot0 \times 10^{-8}$	Kuenen (1956)

data for stream gravels, that sorting and abrasion may dominate in streams shorter than about 80 km and longer than 240 km, respectively.

Significance of bed sediment lithology

Particle size diminution data are available for different rock types in three gravel bed rivers, the Noe, Clutha and Colorado (Table I). The data from each river cover from 0·26 to 0·53 orders of magnitude of α, equivalent to up to about half the range of the entire data set (Figure 1). Hence the bed sediment lithology may have a very significant influence on α. However, it may be exercised indirectly via related factors such as bed particle shapes and size distributions.

On the basis of their lengths, sorting is probably dominant in the Noe, and abrasion in the Clutha and Colorado. The α data for the Colorado and Clutha gravels (Table I) are reasonably consistent with the corresponding α_1 data (Table II) and hence with the dominance of abrasion in these streams. The α value for shale in the Noe is also reasonably consistent with the corresponding α_1 value in Table II given that the latter is a high value derived from a pumping test. However, it is significantly lower than the α value for the more competent, abrasion-resistant sandstone in the same river. This is consistent with the dominance of sorting, although it is also at least partly attributable to significant lateral inputs of fresh shale, but not sandstone, over the length of the stream (Knighton, 1982).

Significance of the worldwide correlation

It is remarkable that the worldwide data set as a whole (Figure 1) conforms to Equation 4, which applies to individual streams or stream segments, as no discordant data were discarded in the analysis. The wide range of hydraulic parameters of all kinds associated with the disparate flows included in the data set strongly suggests that this result has arisen from mechanisms of hydraulic sorting (Seal *et al.*, 1997) and abrasion (Sternberg, 1875) common to all mobile bed subaerial aqueous flows in which the bed particle size varies exponentially with distance along the stream.

On the basis of a comparable worldwide correlation for the transport of sediments by natural subaerial aqueous flows, Bagnold (1986) reached a similar conclusion. However, his comment that much of the scatter of his data (also about one order of magnitude) may be due to errors of measurement is unlikely to be true in the present case in which each data point itself represents a strong correlation. Here, although the different sampling techniques used by the numerous different researchers represented in the data set may have had a significant effect (Seal *et al.*, 1997), the width of the data band is mostly attributable to real physical phenomena. These may include the bed sediment lithology and particle size distribution, the solids concentration, the discharge, and the relative hydraulic roughness (D divided by the water depth) of the flow. Given the well defined bounds to the data (Figure 1), it is unlikely that underestimated L values had a significant effect.

The suggestion that common mechanisms underlie particle size diminution in mobile bed subaerial aqueous flows of all kinds implies that theoretical models developed for one such kind of flow may also be applicable to others. This has already been shown to be true in some cases. For example, a theoretical model of exponential particle size diminution on mine tailings deltas (Morris, 1993; Morris and Williams, 1997d) has proven to be applicable to mine co-disposal deltas (Morris and Williams, 1997a, c) and, after the incorporation of abrasion phenomena, to natural streams and alluvial fans (Morris and Williams, 1997b). Many natural and artificial flows readily meet the conditions necessary for this model to apply. However, departures from the model such as unsteady flow, unusual sediment particle size distributions, the confluence of streams, and controls such as rock bars also occur frequently, restricting its applications.

CONCLUSIONS

It has been shown that there is a very strong worldwide correlation between the exponential sorting coefficient, α, for subaerial aqueous flows of all kinds and the stream length, L (Figure 1). This correlation extends over virtually the whole range of L, stream sediment particle sizes, and other hydraulic conditions found on Earth. It applies irrespective of the solids concentration of the flow, whether the flow or sediment is natural or artificial, or whether normal or reverse sorting occurs. However, the scatter of the data, though relatively small, is significant. It is mostly attributable to real mobile bed flow phenomena rather than to errors of measurement.

A comparison of the worldwide α data with laboratory abrasion coefficient data has suggested that abrasion is unlikely to be significant in gravel and sand bed streams shorter than about 10 km to 100 km and about 500 km, respectively. This is broadly consistent with the conclusions of Morris and Williams (1997b) based on limited field abrasion data from gravel bed streams.

The apparent universality of the worldwide correlation for α for subaerial aqueous mobile bed flows strongly suggests that exponential particle size diminution in all such flows is subject to common underlying mechanisms of hydraulic sorting and abrasion. This is consistent with the earlier successful application, after some modifications and additions, of a theoretical model of particle size diminution on mine tailings deltas (Morris, 1993; Morris and Williams, 1997d) to coal mine co-disposal deltas, natural streams and alluvial fans (Morris and Williams, 1997a, b, c).

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